1.0 INTRODUCTION

The phenomenon of negative downdrag has progressively gained attentions from the engineering profession after many foundation failures due to excessive downdrag. Failures in either serviceability or ultimate limits or both of structures are expected if downdrag is not considered in the foundation design, particularly in settling subsoil.

This note reviews the available design concepts pertaining to this subject, presents a case history on instrumented piles showing negative skin friction and finally demonstrates a simplified design approach using the common commercial spreadsheet, Microsoft Excel. Discussions will be addressed on the load transfer mechanism at the pile/soil interface, determination of maximum pile axial stress and depth of neutral plane, performance of the pile subject to downdrag and parametric study. Methods of reducing downdrag load on piles, like applying slip coat material on piles, providing pile sleeves and other methods will be highlighted.

2.0 NEGATIVE SKIN FRICTION & CAUSES

Negative skin friction (NSF) is in fact a downward friction imposed on foundation piles as a result of subsoil settlement. NSF is usually mobilized to ultimate stress limit in most cases except at very close proximity to the neutral plane as discussed later. It needs only few millimeters of relative displacement between the settling subsoil and the pile shaft surface, which is not uncommon to have relative displacement at the pile-soil interface more than these values in normal subsoil settlement problem, to fully mobilise the shaft resistance in either upward or downward directions.

There are five probable, but not limited to, reasons of existence of NSF, namely,

a. Self-weight of unconsolidated recent fill,
b. Surcharge-induced consolidation settlement
c. Consolidation settlement after dissipation of excess pore pressure induced by pile driving,
d. Lowering of groundwater level,
e. Collapse settlements due to wetting of unsaturated fill, and
f. Crushing of crushable subsoil under sustained loading, causing subsoil settlement.

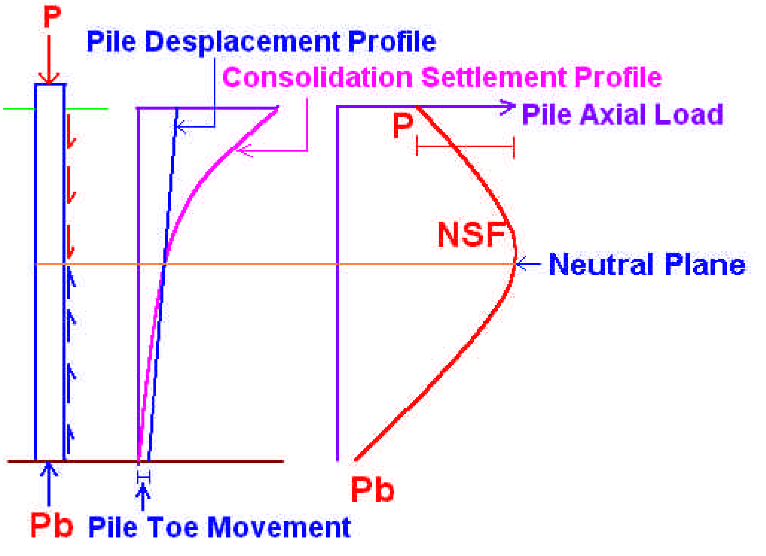

Figure 1 Schematic Diagram of Pile with NSF

3. ANALYTICAL METHODS

There are several ways in assessing pile subjected to negative skin friction. This will, or course, involves in determining the dragload and also the location of the neutral plane in order to assess the pile top movement. Neutral plane represents the mid point of a transition zone or point (depending on pile-soil interface model) where the pile shaft friction gradually changing from negative to positive along the pile shaft. It is also a location on the pile shaft where there is no relative displacement between the pile shaft and the settling subsoil at that point.

The followings are the conventional engineering solutions in tackling the analyses of NFS:

a. Closed Form Equations : These approaches normally involves locating the neutral plane for computation of dragload with balancing the downward loading and the upward resistance. The closed form solutions based on simplified assumptions offer equations to compute the dragload and the position of neutral plane. All the physical load transfer behavours at the pile-soil interface are modelled by simple equations and therefore can be easily processed using mathematical operations. Sometimes, engineer will resort to some numerical iterative computational processes to determine the neutral plane with force equilibrium and therefore the dragload. Such approach is more versatile in dealing with heterogeneous subsoil stratum, varying pile properties and possibility of modeling of slippage and non-linear load transfer behaviour at the pile-soil interface. For the load transfer behaviour, the commonly used models are rigid-plastic model, elastic–plastic model and hyperbolic model.

b. Continuum Approaches : This usually can only offer solution to very ideal subsoil condition. For practical use, simplification of the input parameters are required.

The magnitude of NSF is a function of the ultimate resistance and the relative displacement between soil and pile shaft interface, and also the pile toe stiffness. To accurately compute NSF, the subsoil displacement profile, structural pile shaft stiffness and the pile toe stiffness are required to determine the neutral plane.

The cumulative pile shaft resistance above the neutral plane is a dragload whereas positive resistance is found below the neutral plane to resist the total downward loads from NSF and the imposed load at pile top. Maximum pile axial compression is located at the neutral plane. The conventional rigid-plastic model tends to over-predict NSF as it does not consider load transfer behaviour and structural stiffness or compressibility of pile element. The derivation for computation of dragload and the location of neutral plane based on the rigid-plastic soil model is given in Appendix 1. Figure 1 shows the schematic diagram of the pile subjected to NSF.

Displacement profile of the compressible or settling subsoil shall be determined in order to compute the relative displacement between the pile shaft and the embedded subsoil and therefore the degree of mobilization of the unit shaft friction for force equilibrium. Despite it is less sensitive in the dragload computation, the pile settlement behaviour are very sensitive to subsoil displacement profile. In normal cases, the displacement profile of consolidating subsoil shows a concave profile, which is fairly different to the linear profile as assumed in most closed form solutions or continuum approaches.

Shaft Resistance (R_{su})

The following four methods are usually used to determine the ultimate shaft resistance:

a. Total Stress Approach (α-Method) : $\tau_{ult} = \alpha \times C_u$
b. Effective Stress Approach (β-Method) : $\tau_{ult} = K \times \sigma_v' \times \tan\delta' = \beta \times \sigma_v'$
c. In-Situ Test Results (SPT-N or CPT) : $\tau_{ult} = X \times$ SPT-N or $f(q_u, f_s)$
d. High Strain Dynamic Pile Test : τ_{mob} derived from wave analyses

In normal case, the β-method is the more relevant approach as effective stress approach is often associated with consolidation process of the subsoil.

There are always disputable arguments on whether the unit shaft resistance is the same for both the upward and downward resistance at the same depth. Some researchers advocate that the confining stiffness in the embedded subsoil and the vertical effective stress will be reduced in the event of upward resistance from the pile to the soil and therefore reducing the unit shaft resistance. From the logical point of view, such arguments have the supporting evidence from the model tests. In the pile group model test in consolidating subsoil, the "*hanging effect*" of the centre pile due to the support from the perimeter piles is obvious. Therefore, the normal way of computing the unit shaft resistance based on the effective overburden stress with the lateral earth pressure coefficient is not appropriate for the centre pile. Nevertheless, the assumption of same positive and negative resistance is a conservative approach and will not lead to a unsafe pile design.

Toe Resistance (R_{tu})

The following four methods are usually used to determine the ultimate toe resistance:

a. Total Stress Approach : $R_{tu} = A_t \times X\alpha \times C_u$
b. Effective Stress Approach : $R_{tu} = A_t \times N_t \times \sigma_{z=D}'$
c. In-Situ Test Results (SPT-N or CPT) : $R_{tu} = A_t \times X \times$ SPT-N or $f(q_u, f_s)$
d. High Strain Dynamic Pile Test : R_{tu} derived from wave analyses

4.0 LOADING NATURE & TIMING OF IMPOSED LOAD

The nature and timing of the imposed loading at pile top will also affect the magnitude of NSF. Subsequent imposed loads after development of NSF will further penetrate the pile into the soil and therefore reduce or even eliminate the NSF developed earlier. This will also result in higher neutral plane, which corresponds to less NSF but more settlement on pile. Live load, transient load and cyclic load fall into this category of load, which reduces the NSF developed after exertion of dead load or sustained load on pile. Due consideration should be given to this aspect to avoid over-conservatism in pile design.

5.0 SAFETY FACTOR & SERVICEABILITY LIMIT

In the pile design with NSF, the maximum pile axial compression (allowable pile top load plus NSF) should not exceed the allowable structural capacity of the pile. There are two design considerations where safety factor is applied in a different way. If the pile is physically sleeved from NSF, than safety factor of two against ultimate upward resistance from the pile shaft below neutral plane is normally applied, which is similar to the normal pile design. This proposed safety factor is to have some control on the serviceability limit of the pile. If the pile is slip coated, then constant NSF of very much reduced magnitude should be taken as part of the imposed load in additional to the pile top load and the same safety factor can be applied.

In term of safety of factor on geotechnical pile capacity, full shaft resistance along the entire pile penetration length should be used to compute the ultimate geotechnical pile capacity. The reason being is that the prerequisite of a pile achieving the ultimate condition in term of geotechnical capacity is to continue settling at a limiting load. In such event, the negative skin friction will no more exist, but with the price of larger or even excessive settlement. Therefore, negative skin friction shall not be considered concurrently with the ultimate limit state of the pile.

6.0 DESIGN CONDITIONS

Due to different subsoil conditions, there are two major pile design conditions as follows:

a. Frictional Pile
- Higher neutral plane
- Lesser dragload on piles
- Larger foundation settlement (Serviceability to be checked)

b. End-Bearing Pile
- Lower neutral plane
- Larger dragload on piles
- Lesser foundation settlement (Safety factor on pile structure to be checked)

7.0 SINGLE PILE & GROUP PILES

There are fundamental differences of pile design with NSF in the conditions of single pile and group piles. In single pile situation, the dragload is primarily controlled by the free field subsoil settlement profile and the degree of mobilisation of the pile shaft resistance to its ultimate limit. Whereas, in the situation of group piles, the stiffer group piles will disturb the free field settlement profile and there will be a “hang-up” effect (interior piles will have less NSF and external piles will have relative more NSF as a single pile). Table 1 shows the summary of useful expressions for computing NSF at various locations of group piles. In any cases, the total NSF imposed on the pile or group piles should not be greater than the total imposed fill weight inducing the subsoil settlement within the effective coverage of the pile or group piles.

Table 1 Summary of NSF of Group Piles

Pile Location	Briaud et al (1991)	Broms (1976)	Combarieu (1974)
Corner	$\frac{3}{4}\times F_{n(\infty)}$	$S\times L\times S_u + q\times(S\times L/4+L^2/16)$	$7/12\times F_{n(S)}+5/12\times F_{n(\infty)}$
Edge	$\frac{1}{2}\times F_{n(\infty)}$	$S\times L\times S_u + q\times S\times L/4$	$5/6\times F_{n(S)}+1/6\times F_{n(\infty)}$
Interior	$q\times S^2$	$q\times S^2$	$F_{n(b)}$

Notes: S : Pile Spacing L : Depth of Neutral Plane
q : Increase of Vertical Effective Stress in Subsoil
S_u : Undrained Shear Strength
$F_{n(\infty)}$: NSF in Single Pile (needs separate analysis)
$F_{n(S)}$: NSF for Pile with spacing of S (needs separate analysis)

8.0 PILE TESTING REQUIREMENTS

In the testing of pile with NSF, there are some special requirements in order to properly verify the design:

a. Sleeve the pile shaft above predetermined neutral plane to verify the positive contribution to the pile capacity.
b. Increase test load to two times of the total estimated downward imposed loads (allowable pile top load plus NSF). This test load is to overcome the positive shaft resistance above the neutral plane during testing and simulate NSF in service condition, and then test the positive resistance of the test pile below the neutral plane with safety factor of two.
c. Instrumented test pile or use of high strain dynamic pile test to monitor both the load-transfer behaviour along the pile shaft and at the pile toe.

9.0 PREVENTIVE MEASURES

Some preventive measures can be considered to overcome the engineering problems generated by NSF. There are:

a. Preload the subsoil to reduce or eliminate subsoil settlement by surcharge before pile installation.
b. Sleeve or isolate the pile shaft from the surrounding settling soil throughout the portion where the NSF exists or to the depth of the estimated neutral plane.
c. Application of slip coating (creeping interface medium) on pile shaft.
d. Reserve structural pile capacity for NSF by downgrading the pile top load.
e. Adopt frictional piles with recognition of larges settlement in both the foundation and superstructure designs.

The following design considerations shall be taken in designing of slip coating using bitumen or other creeping interface materials, which will significantly affect its effectiveness :

a. Thickness of the slip coating material,
b. Rate of Shearing or rate of consolidation settlement,
c. Viscosity of the coating materials
d. Ambient temperature of subsoil.
e. Protection of the coating materials during handling and installation.

10.0 CASE STUDY

A case study of two numbers of 35m long instrumented abutment piles (ϕ500mm) installed in granitic residual soil (SPT-N = 5 to 35) was presented. In this case history, the neutral planes are located at the depth of about 7m to 10m. The measured NSF ranges from 920kN (middle piles) to 1100kN (edge pile) even in a reasonably good residual soil with 65mm surface settlement under the embankment loading of 8m high. The details of thus case history can be referred in a technical paper as attached in Appendix 2.

Findings and Conclusions

The following summarised highlights can be drawn:

a. NSF does exist in reasonably good residual soil even there is only slight subsoil settlement under loading.
b. Conventional rigid-plastic soil model tends to over-predict NSF, whereas the elastic-plastic soil model is an improved methodology over rigid-plastic model. If load transfer behaviour and stiffness of pile element are considered using numerical iterative process, the NSF prediction will be more accurate.
c. Pile group effect in NSF design is significant and should not be neglected for an optimised design.
d. Due allowance on pile capacity downgrading or measures to reduce NSF should be considered.
e. More researches on NSF are needed, particularly for the group piles.

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APPENDIX 1

ANALYTICAL MODELLING OF NEGATIVE SKIN FRICTION

A. Rigid-Plastic Soil Model :

Assumptions :

a. Negative, positive shaft resistances and toe resistance are all fully mobilised to ultimate condition.
b. The unit shaft and toe resistances are linearly increased with depth.
c. Positive (r_s) and negative (q_n) unit shaft friction is the same at the same depth, i.e. $q_n = az = r_s = bz$;

$$Q_n = \frac{A_s \cdot a \cdot Z_{NP}^2}{2} \qquad \text{----- Eq. (1)}$$

$$R_{su} = \frac{A_s \cdot a \cdot D^2}{2} \qquad \text{----- Eq. (2)}$$

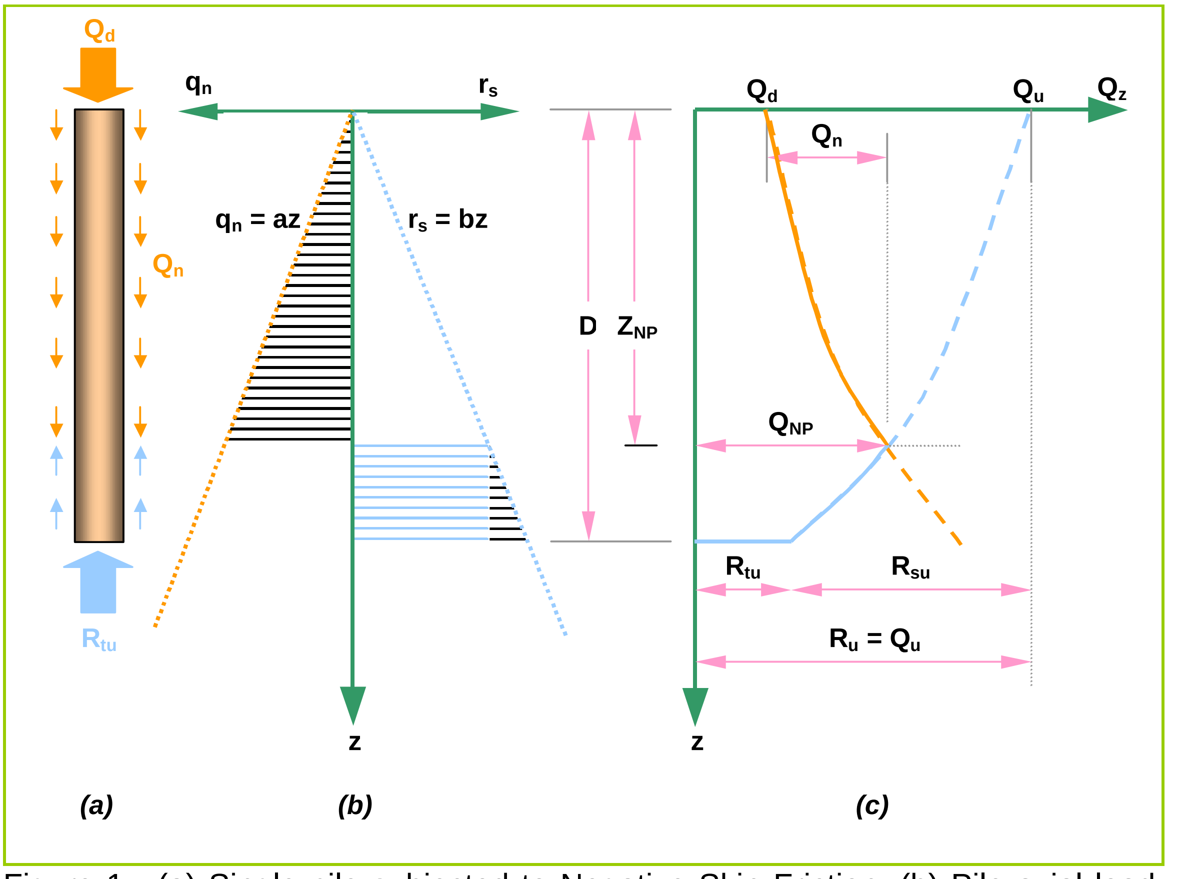

Figure 1 : (a) Single pile subjected to Negative Skin Friction. (b) Pile axial load distribution. (c) Distribution of positive and negative unit shaft resistance, r_s and q_n respectively on pile shaft.

Definitions:

D = Pile Penetration Length
B = Pile Diameter
Z_{NP} = Depth of Neutral Plane
q_n = Negative Unit Shaft Resistance = az (Linearly increasing with depth)
r_s = Positive Unit Shaft Resistance = bz (Linearly increasing with depth)
Q_d = Imposed Load at Pile Top
Q_n = Negative Skin Friction on Pile above Neutral Plane
= $\int_0^{Z_{NP}} A_s \beta_n \sigma_z' dz = \int_0^{Z_{NP}} A_s K_{sn} \sigma_z' \tan\delta \cdot dz$ (β refers to Table 1)
Q_{NP} = Pile Axial Load at Neutral Plane
R_{tu} = Ultimate Pile Toe Resistance
= $A_t N_t \sigma_{z=D}'$ (N_t refers to Table 1) or $A_t \{c' N_c + \sigma_{z=D}' N_q\}$ (N_c & N_q refers to NAVFAC, 1982)
R_{su} = Ultimate Pile Shaft Resistance over Whole Shaft Length
= $\int_0^{D} A_s \beta_p \sigma_z' dz = \int_0^{D} A_s K_{sp} \sigma_z' \tan\delta \cdot dz$ (β refers to Table 1)
Q_u = R_u = Ultimate Pile Capacity = R_{tu} + R_{su}

Table 1 Ranges of φ, β and N_t values

Soil Type	φ (Degree)	β	N_t
Clay	25 ~ 30	0.25 ~ 0.35	3 ~ 30
Silt	28 ~ 34	0.27 ~ 0.50	20 ~ 40
Sand	32 ~ 40	0.30 ~ 0.60	30 ~ 150
Gravel	35 ~ 45	0.35 ~ 0.80	60 ~ 300

Note : Ontario Highway Bridge Design Code (1992)

Two dimensionless parameters are introduced as follows:

a. Ratio of Ultimate Pile Capacity to Ultimate Shaft Resistance, α :

$$\alpha = \frac{R_u}{R_{su}}$$

b. Factor of Safety on Pile Capacity against Ultimate Pile Capacity, F_s :

$$F_s = \frac{R_u}{Q_d}$$

Normalised Depth of Neutral Plane to Pile Penetration Length using equilibrium equation, Eq. (3) below for the forces exerted on pile shaft above and below the Neutral Plane and also the pile toe:

$$Q_{NP} = Q_d + \int_0^{Z_{NP}} A_s q_n dz = R_{tu} + \int_{Z_{NP}}^{D} A_s r_s dz \qquad \text{------ Eq. (3)}$$

$$\Rightarrow Q_d + A_s \cdot a \cdot \left.\frac{z^2}{2}\right|_0^{Z_{NP}} = R_{tu} + A_s \cdot a \cdot \left.\frac{z^2}{2}\right|_{Z_{NP}}^{D}$$

$$\Leftrightarrow Q_d + \frac{A_s \cdot a \cdot Z_{NP}^2}{2} = R_{tu} + \frac{A_s \cdot a \cdot (D^2 - 2Z_{NP}^2)}{2}$$

$$\Leftrightarrow \frac{2(Q_d - R_{tu})}{A_s \cdot a} = D^2 - 2Z_{NP}^2$$

$$\Rightarrow \frac{Z_{NP}}{D} = \sqrt{\left[\frac{1}{2} - \frac{(Q_d - R_{tu})}{2 \cdot \frac{D^2 \cdot A_s \cdot a}{2}}\right]} = \sqrt{\left[\frac{1}{2} - \frac{(Q_d - R_{tu})}{2 \cdot R_{su}}\right]} = \sqrt{\left[\frac{1}{2}\left(\frac{R_{su} - Q_d + R_{tu}}{R_{su}}\right)\right]}$$

$$\Leftrightarrow \frac{Z_{NP}}{D} = \sqrt{\left[\frac{1}{2}\left(\frac{R_u - \frac{R_u}{F_s}}{R_{su}}\right)\right]} = \sqrt{\frac{a}{2}\left[1 - \frac{1}{F_S}\right]} \qquad \text{------ Eq. (4)}$$

Normalised Dragload at Neutral Plane to Ultimate Pile Capacity using Eq. (3) :

$$Q_{NP} = Q_d + \frac{A_s \cdot a \cdot Z_{NP}^2}{2} = \frac{R_u}{F_s} + \frac{A_s \cdot a \cdot D^2 \cdot \left(\frac{Z_{NP}^2}{D^2}\right)}{2} = \frac{R_u}{F_s} + R_{su} \cdot \frac{a}{2} \cdot \left[1 - \frac{1}{F_s}\right] = \frac{R_u}{F_s} + \frac{R_u}{2}\left[1 - \frac{1}{F_s}\right]$$

$$\Rightarrow \frac{Q_{NP}}{R_u} = \frac{1}{2}\left(1 + \frac{1}{F_s}\right) \qquad \text{------ Eq. (5)}$$

B. Elastic-Plastic Soil Model :

Assumptions:

a. The pile is rigid and incompressible.
b. The subsoil settlement profile is linear with maximum at the ground surface and decreasing with depth.
c. Negative and positive shaft resistances are all fully mobilised to ultimate condition except at the transition zone at the neutral plane.

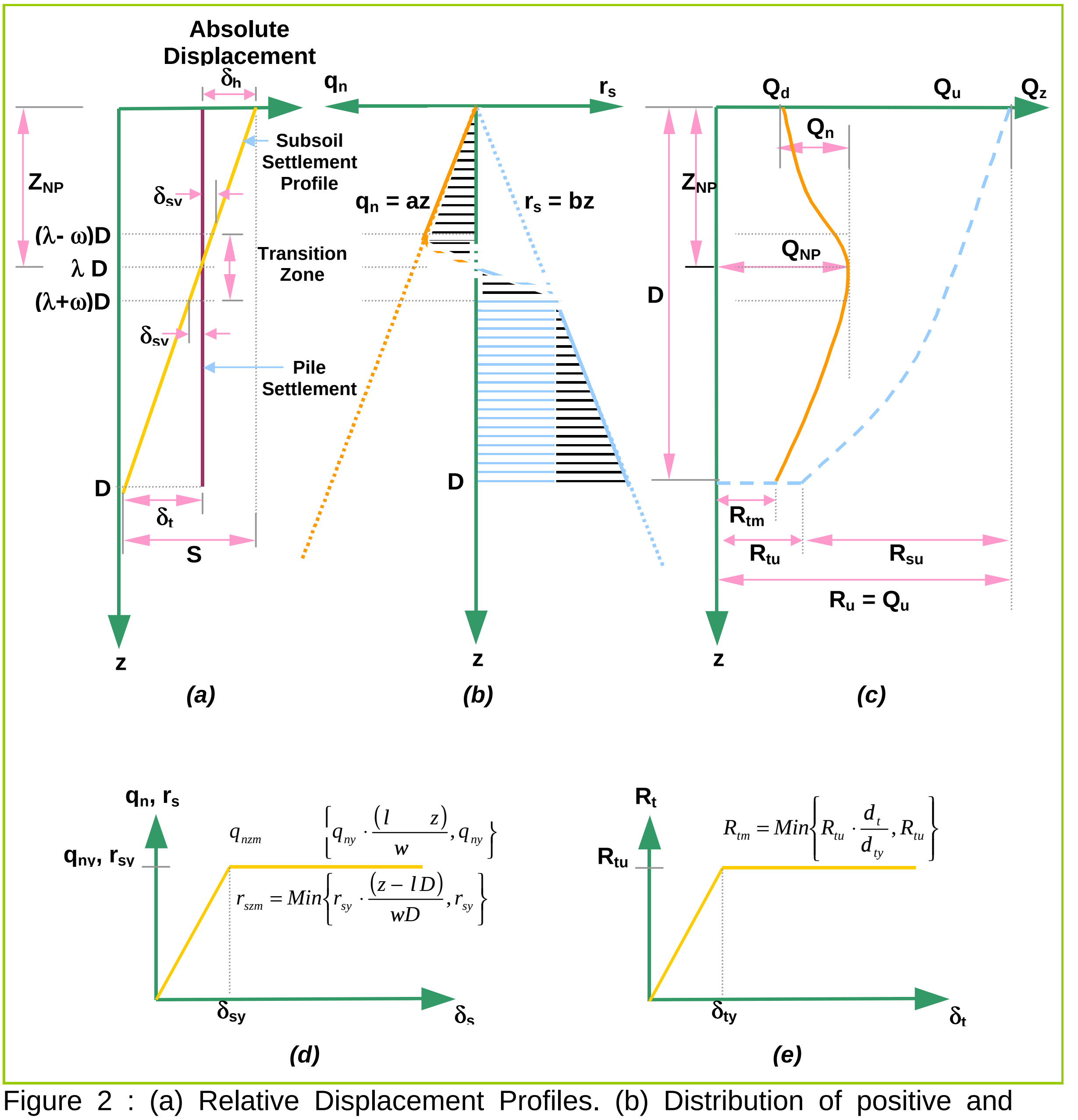

Figure 2 : (a) Relative Displacement Profiles. (b) Distribution of positive and negative unit shaft resistance, r_s and q_n respectively on pile shaft. (c) Pile axial load distribution. (d) Mobilised Shaft Resistance. (e) Mobilised Toe Resistance.

Three dimensionless parameters are introduced as follows:

$\Psi = \dfrac{d_{ty}}{S}$ where δ_{ty} is the relative displacement between the pile toe and the soil at the pile toe that is required to yield the toe resistance.

$\omega = \dfrac{d_{sy}}{S}$ where δ_{sy} is the relative displacement between the pile and the soil around the pile shaft that is required to yield the shaft resistance.

$\lambda = \dfrac{d_h}{S} = \dfrac{Z_{NP}}{D}$ where δ_h is the relative settlement between the pile head and the settled ground surface.

Subscripts s, t, y and m denote the shaft, toe, yield and mobilised respectively.

S denotes the total compression of the subsoil within the pile penetration length, D by integrating the vertical strain in the said subsoil strata. In this model, the vertical strain is identical at every level within the subsoil.

δ_t is the relative settlement between the pile toe and the settled subsoil at the pile toe.

Normalised Depth of Neutral Plane to Pile Penetration Length using equilibrium equation, Eq. (6), below for the forces exerted on pile shaft above and below the Neutral Plane and also the pile toe:

$$Q_{NP} = Q_d + \int_0^{(l-w)D} A_s q_{ny}\, dz + \int_{(l-w)D}^{lD} A_s q_{nzm}\, dz = \int_{lD}^{(l+w)D} A_s r_{szm}\, dz + \int_{(l+w)D}^{D} A_s r_{sy}\, dz + R_{tm} \quad \text{------ Eq. (6)}$$

$$\Rightarrow Q_d + \int_0^{(l-w)D} A_s \cdot a \cdot z \cdot dz + \int_{(l-w)D}^{lD} A_s \cdot a \cdot z \cdot \frac{(lD - z)}{wD} \cdot dz$$

$$= \int_{lD}^{(l+w)D} A_s \cdot a \cdot z \cdot \frac{(z - lD)}{wD} \cdot dz + \int_{(l+w)D}^{D} A_s \cdot a \cdot z \cdot dz + R_{tu} \cdot \frac{d_t}{d_{ty}}$$

$$\Leftrightarrow Q_d + A_s \cdot a \cdot \left.\frac{z^2}{2}\right|_0^{(l-w)D} + A_s \cdot a \cdot \left.\left(\frac{lz^2}{2w} - \frac{z^3}{3wD}\right)\right|_{(l-w)D}^{lD} = A_s \cdot a \cdot \left.\left(\frac{z^3}{3wD} - \frac{lz^2}{2w}\right)\right|_{lD}^{(l+w)D}$$

$$+ A_s \cdot a \cdot \left.\frac{z^2}{2}\right|_{(l+w)D}^{D} + R_{tu} \cdot \frac{d_t}{d_{ty}}$$

$$\Leftrightarrow Q_d = -A_s \cdot a \cdot \left.\frac{z^2}{2}\right|_0^{(l-w)D} + A_s \cdot a \cdot \left.\left(\frac{z^3}{3wD} - \frac{lz^2}{2w}\right)\right|_{(l-w)D}^{(l+w)D} + A_s \cdot a \cdot \left.\frac{z^2}{2}\right|_{(l+w)D}^{D} + R_{tu} \cdot \frac{d_t}{d_{ty}}$$

$$\Leftrightarrow Q_d = -\frac{A_s a(l-w)^2 D^2}{2} + A_s \cdot a \cdot \left(\frac{(l+w)^3 D^2 - (l-w)^3 D^2}{3w} - \frac{l(l+w)^2 D^2 - l(l-w)^2 D^2}{2w} \right)$$

$$+ \frac{A_s a(D^2 - (l+w)^2 D^2)}{2} + R_{tu} \cdot \frac{d_t}{d_{ty}}$$

$$\Leftrightarrow \frac{R_u}{F_s} = -(l-w)^2 \cdot R_{su} + \left(\frac{2\left((l+w)^3 - (l-w)^3\right)}{3w} - \frac{l(l+w)^2 - l(l-w)^2}{w} \right) \cdot R_{su}$$

$$+ \left[1 - (l+w)^2\right] \cdot R_{su} + (R_u - R_{su}) \cdot \frac{(S - d_h)}{d_{ty}}$$

$$\Leftrightarrow \frac{R_u}{F_s} = \left(-2l^2 - \frac{2}{3}w^2 + 1 \right) \cdot R_{su} + (R_u - R_{su}) \cdot \frac{(S - lS)}{yS}$$

$$\Leftrightarrow \frac{a}{F_s} = \left(-2l^2 + 1 - \frac{2}{3}w^2 \right) + (a-1) \cdot \frac{(1-l)}{y}$$

$$\Leftrightarrow 2yl^2 + (a-1)l - (a-1) - y\left(1 - \frac{2}{3}w^2 - \frac{a}{F_s}\right) = 0$$

$$\Rightarrow l = \frac{Z_{NP}}{D} = \frac{-(a-1) \pm \sqrt{(a-1)^2 - 4 \cdot 2y \cdot \left(-(a-1) - y\left(1 - \frac{2}{3}w^2 - \frac{a}{F_s}\right)\right)}}{2 \cdot 2y}$$

$$= \frac{\sqrt{(a-1)^2 + 8y(a-1) + 8y^2\left(1 - \frac{2}{3}w^2 - \frac{a}{F_s}\right)} - (a-1)}{4y}$$

$$\Rightarrow l = \frac{Z_{NP}}{D} = \frac{\sqrt{(a-1)^2 + 8y(a-1) + 8y^2\left(1 - \frac{2}{3}w^2 - \frac{a}{F_s}\right)} - (a-1)}{4y} \qquad \text{------ Eq. (7)}$$

Normalised Dragload at Neutral Plane to Ultimate Pile Capacity using Eq. (6) :

$$Q_{NP} = Q_d + \int_0^{(l-w)D} A_s q_{ny} dz + \int_{(l-w)D}^{lD} A_s q_{nzm} dz$$

$$= Q_d + \int_0^{(l-w)D} A_s \cdot a \cdot z \cdot dz + \int_{(l-w)D}^{lD} A_s \cdot a \cdot z \cdot \frac{(lD - z)}{wD} \cdot dz$$

$$= Q_d + A_s \cdot a \cdot \left.\frac{z^2}{2}\right|_0^{(l-w)D} + A_s \cdot a \cdot \left.\left(\frac{lz^2}{2w} - \frac{z^3}{3wD} \right)\right|_{(l-w)D}^{lD}$$

$$= Q_d + \frac{A_s a(l-w)^2 D^2}{2} + A_s \cdot a \cdot \left(\frac{l^3 D^2 - l(l-w)^2 D^2}{2w} - \frac{l^3 D^2 - (l-w)^3 D^2}{3w} \right)$$

$$= \frac{R_u}{F_s} + R_{su} \cdot \left[(l - w)^2 + \left(\frac{l^3 - l(l-w)^2}{w} - \frac{2\left(l^3 - (l-w)^3\right)}{3w} \right) \right]$$

$$= \frac{R_u}{F_s} + R_{su} \cdot \left[l^2 - 2wl + w^2 + \left(\frac{l^3 - l\left(l^2 - 2wl + w^2\right)}{w} - \frac{2\left(l^3 - (l-w)^3\right)}{3w} \right) \right]$$

$$= \frac{R_u}{F_s} + R_{su} \cdot \left[l^2 - 2wl + w^2 + \left(2l^2 - wl - 2l^2 + 2wl - \frac{2w^2}{3} \right) \right]$$

$$= \frac{R_u}{F_s} + R_{su} \cdot \left[l^2 - 2wl + w^2 + \left(wl - \frac{2w^2}{3} \right) \right]$$

$$= \frac{R_u}{F_s} + R_{su} \cdot \left[l^2 - wl + \frac{w^2}{3} \right]$$

$$\Rightarrow \frac{Q_{NP}}{R_u} = \frac{1}{F_s} + \frac{1}{a} \cdot \left(l^2 - wl + \frac{w^2}{3} \right) \qquad \text{------ Eq. (8)}$$

As shown in Figure 2, the thickness of the transition zone is :

$$t_{trans} = (l - w)D - (l + w)D = 2wD = 2\frac{d_{sy}}{S}D$$

The above expression implies that the thickness of the transition zone, t_{trans}, will reduce when the stiffness of the shaft resistance increases (δ_{sy} reduces) and/or compressibility of the soil increases (S increases).

Eq. (7) and (8) will only be valid when the following conditions are satisfied;

a. Transition zone is fully contained within the length, D, of the pile penetration inclusively.

Upper limit : $(\lambda - \omega)\ D \geq 0 \Rightarrow (\lambda - \omega) \geq 0$
Lower limit : $(\lambda + \omega)\ D \leq D \Rightarrow (\lambda + \omega) \leq 1$

b. For the valid expression of mobilised toe resistance, $R_{tm} = R_{tu}\frac{d_t}{d_{ty}}$, toe movement, δ_t, shall be within the movement, δ_{ty}. i.e. $\delta_t \leq \delta_{ty}$

$$S = d_t + d_h \Leftrightarrow d_t = S - d_h$$

$$\because d_t \leq d_{ty} \Rightarrow S - d_h \leq d_{ty} \Leftrightarrow \frac{d_h}{S} + \frac{d_{ty}}{S} \geq 1 \Leftrightarrow l + y \geq 1$$

Rigid-Plastic Soil Model is a special case of Elastic-Plastic Soil Model, in which if the following conditions are satisfied, then Eq. (7) and (8) in the Elastic-Plastic Soil Model will become Eq. (4) and (5) in the Rigid-Plastic Soil Model.

a. Full toe resistance is mobilised with the toe movement reaching yield toe resistance.

$d_t = d_{ty}$ and $S = d_t + d_h \Leftrightarrow l + y = 1$

b. When the stiffness of the shaft resistance becomes infinite,

$d_{sy} = 0 \Rightarrow w = 0$

C. Numerical Iterative Approach

Hyperbolic Soil Model

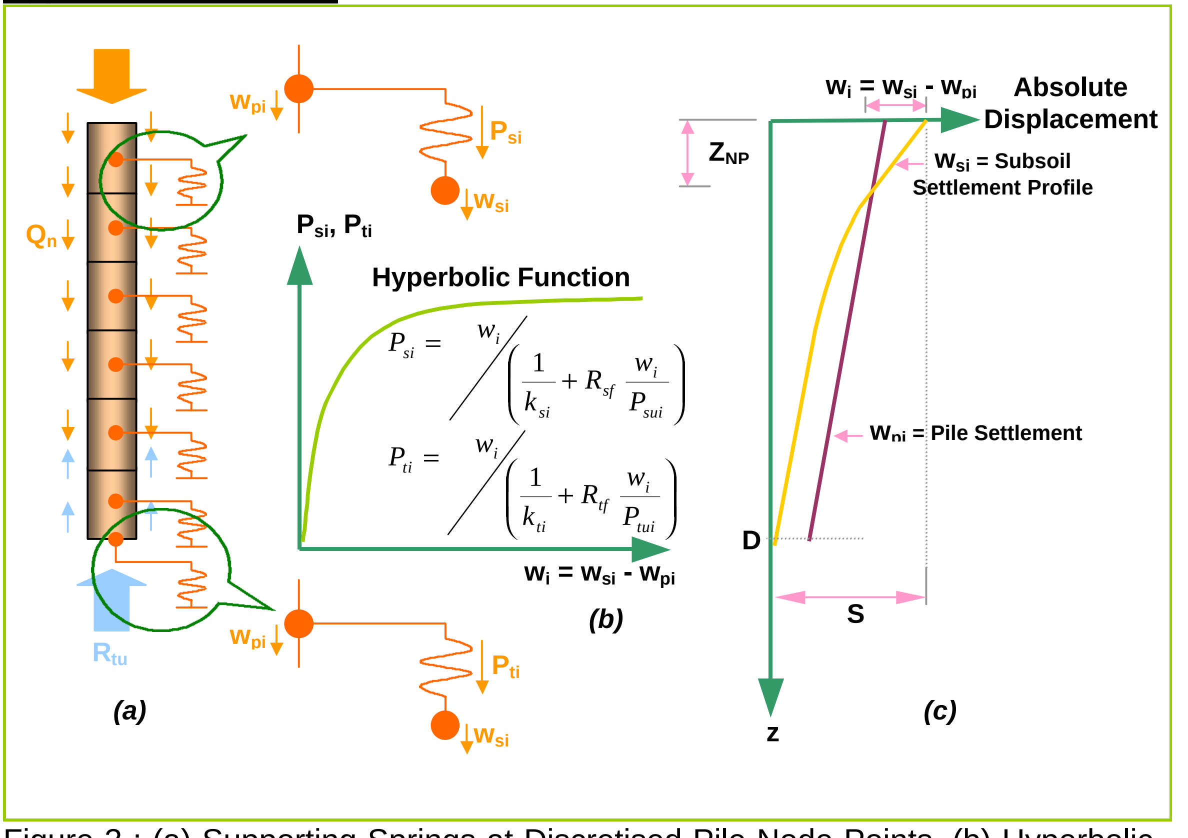

Figure 3 : (a) Supporting Springs at Discretised Pile Node Points. (b) Hyperbolic Function of the Supporting Springs. (c) Displacement Profiles of Pile and Soil.

Hyperbolic Function for pile axial load at Node I :

$$P_i = \frac{w_i}{\left(\frac{1}{k_i} + R_f \frac{w_i}{P_{ui}}\right)} \qquad \text{------ Eq. (9)}$$

Two dimensionless parameters, namely $\overline{w_i}$ and $\overline{P_i}$;

$$\Rightarrow \overline{w_i} = \frac{\overline{P_i}}{\left(1 - \overline{P_i}\right)} \qquad \text{where } \overline{P_i} = R_f \frac{P_i}{P_{ui}}; \overline{w_i} = R_f \frac{w_i}{w_{ui}}; w_{ui} = \frac{P_{ui}}{k_i}$$

To derive tangential stiffness of the hyperbolic function, differentiate equation Eq. (9) to get;

$$\frac{dP_i}{dw_i} = \frac{k_i}{\left(1 + \overline{w_i}\right)^2} \qquad \text{------ Eq. (10)}$$

Pile Shaft

Initial Stiffness at Node i of Pile Shaft :

Randolph & Wroth (1978) : $$k_{si} = 2p \cdot G_i \frac{l_i}{Ln\left({r_m}/{r_o} \right)}$$

where

G_i	:	Soil shear modulus
l_i	:	Pile segment length
r_m	:	Limit radius
r_o	:	Pile radius
L	:	Pile length
n'_s	:	Poisson's ratio

$r_m = 2r \cdot (1 - u'_s)L$ for Gibson soil model (Increasing stiffness with depth)

$$r = \frac{G_{L/2}}{G_L}$$

Shaft Resistance : $$P_i = \frac{w_i}{\left(\frac{1}{k_{si}} + R_{sf} \frac{w_i}{P_{sui}} \right)}$$

Pile Toe

Initial Stiffness at Pile Toe :

Randolph & Wroth (1978) : $$k_{si} = \frac{4G_i r_o \cdot}{(1 - u'_s)}$$

Toe Resistance : $$P_i = \frac{w_i}{\left(\frac{1}{k_{ti}} + R_{tf} \frac{w_i}{P_{tui}} \right)}$$

Correlation of Parameters :

Shear Modulus, G_i : $$G_i = \frac{E'_i}{2(1 + u'_s)} = \frac{E_i}{2(1 + u_s)}$$

For Clay, $$G_i = \frac{E'_i}{2(1 + u'_s)} = \frac{E_i}{2(1 + u_s)} = \frac{2E_{50}}{2(1 + 0.5)} = \frac{2E_{50}}{3}$$

where E'_i : Drained initial tangent modulus
E_i : Undrained initial tangent modulus (=2E_{50} by Duncan et al. 1980)
E_{50} : Secant deviatoric modulus at 50% failure stress (refer to Figure 4)
n'_s : Drained Poisson's ratio
n_s : Undrained Poisson's ratio (=0.5)

For Sand, $E'_i = KP_a\left(K_o \dfrac{s'_v}{P_a}\right)^n$ as recommended by Duncan et al., 1980)

where K : Modulus number (refer to Table 2)
n : Modulus exponent (refer to Table 2)
P_a : Atmospheric pressure

Coefficient of Earth Pressure at rest, K_o (Mayne & Kulhawy, 1982) :

$$K_o = (1 - Sin f')OCR^{Sin f'}$$

Poisson's ratio, n'_s :

For Sand, $u'_s = \dfrac{K_o}{(1+K_o)}$ based on theory of elasticity

For Clay, refer to Table 4

Table 2 Recommended Parameter for Sand

Soil Consistency	D_r (%)	K_s	K	n
Very loose	0 ~ 15	0.6 ~ 1.0	250	0.7
Loose	15 ~ 35	1.0 ~ 1.4	500	0.7
Medium Dense	35 ~ 65	1.4 ~ 1.6	1000	0.7
Dense	65 ~ 85	1.6 ~ 2.0	1500	0.7
Very Dense	85 ~ 100	2.0 ~ 2.4	2000	0.7

Table 3 Recommended Interface Friction Angle for Sand (Kulhawy, 1984)

Pile Material	δ (Degrees)
Rough Concrete	ϕ'
Smooth Concrete	$0.8\phi' \sim \phi'$
Steel	$0.5\phi' \sim 0.9\phi'$
Timber	$0.8\phi' \sim 0.9\phi'$

Table 4 Recommended Poisson's Ratio for Clay (Puolos & Davis, 1980)

Soil Consistency	υ'_s
Soft Non Consolidated Clay	0.3 ~ 0.4
Firm Clay	0.2 ~ 0.3
Stiff Over Consolidated Clay	0.1 ~ 0.2

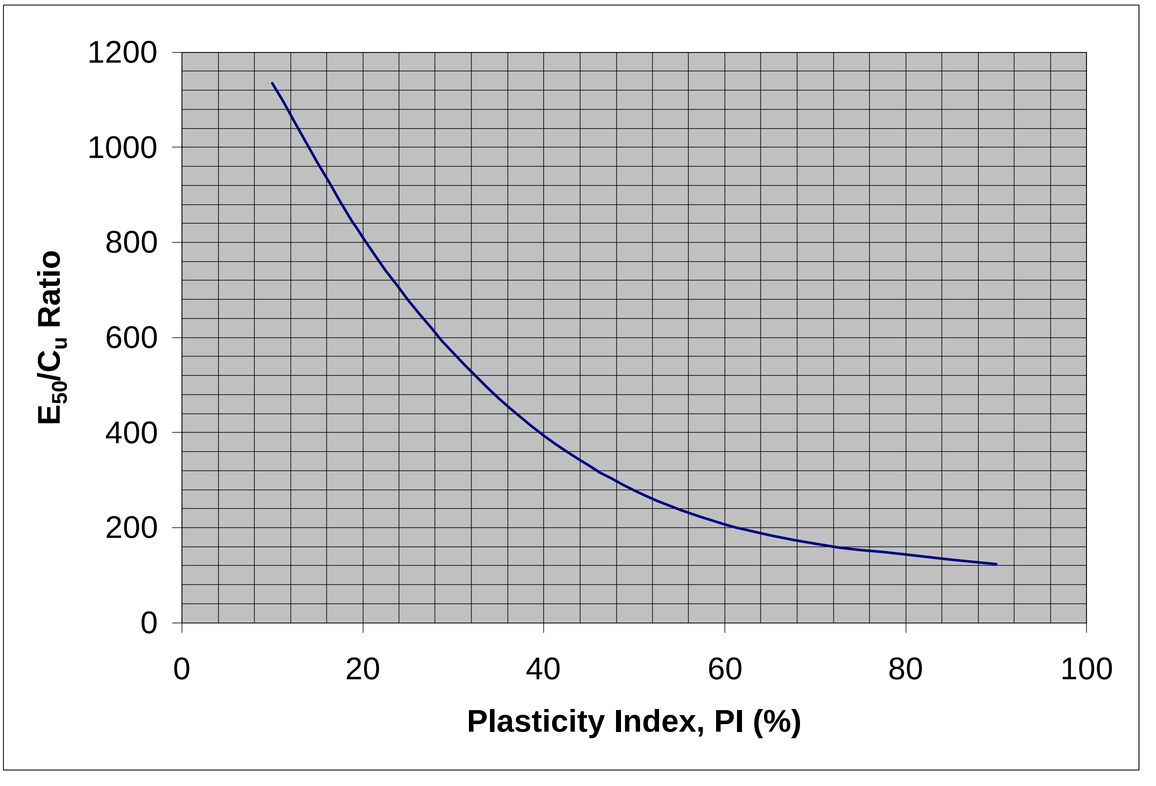

Figure 4 : (a) Ratio of Secant Deviatoric Modulus at 50% Failure Stress to Undrained Shear Strength

The above expressions can be easily implemented in a computer spreadsheet application with an iterative scheme for assessing NSF.

APPENDIX 2

TECHNICAL PAPER